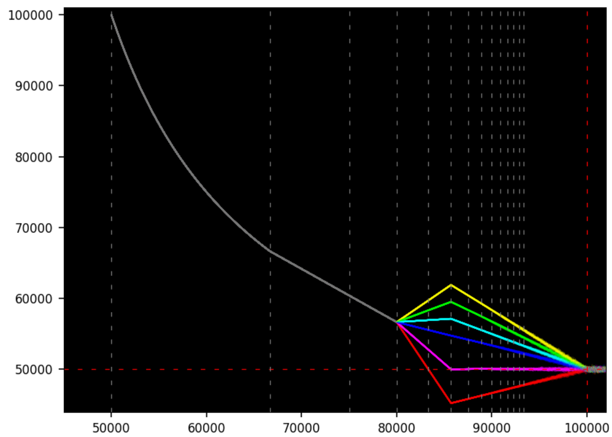
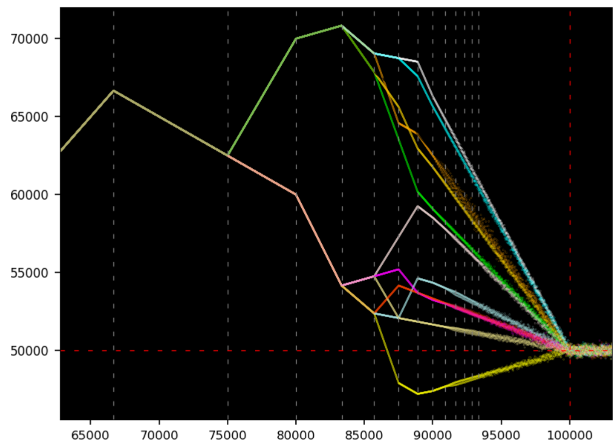
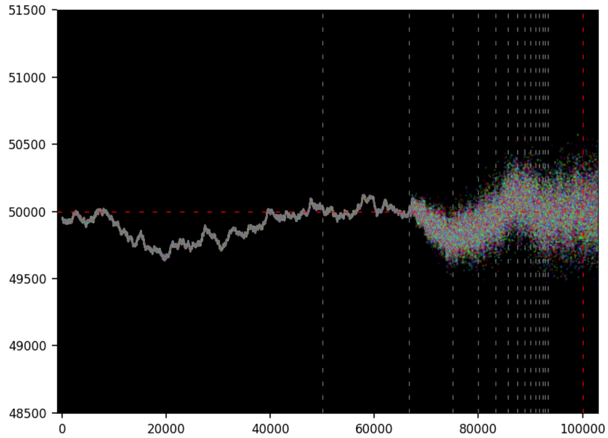
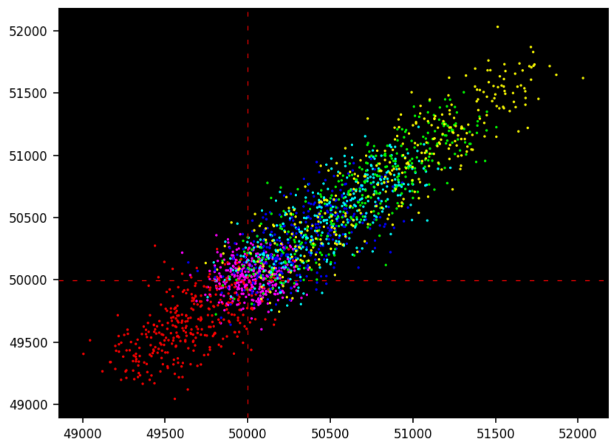
Digit Distribution of Classic Constants

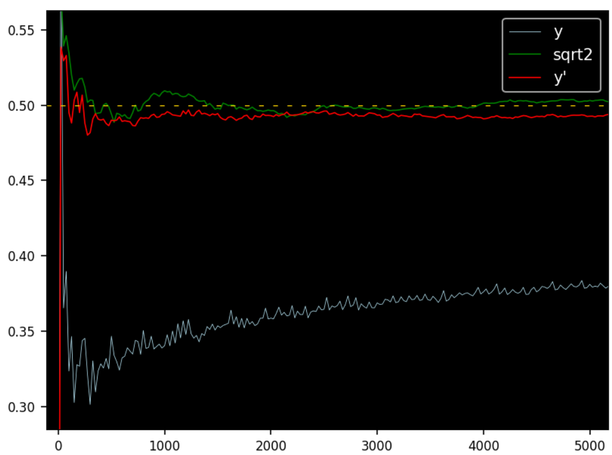
Applications to Fintech, AI, Quantum Systems, Data Synthesis LLMs, Cybersecurity and High-Performance Computing

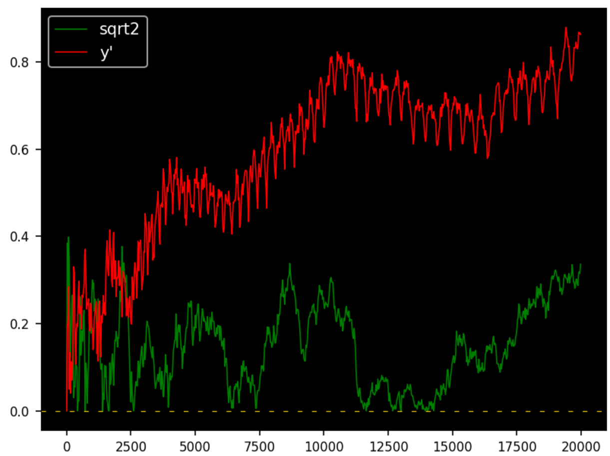












Vincent Granville, Ph.D. | BondingAI.io | Version 4.0, July 2026

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Introduction

Since the first edition entitled "0 and 1 – From Elemental Math to Quantum AI" and released in early 2025, a lot of progress has been made. This fourth edition offers a trip deep into one of the most elusive multi-century old conjectures in number theory: are the binary digits of the fundamental math constants evenly distributed? No one even knows if the proportions of '0' and '1' exist, for any of them: it could oscillate forever between 0% and 100%.

The book includes new chapter 5 on testing randomness with a much simplified version of Weyl's criterion. It also features a breakthrough result regarding the binary digit distribution, stating that the proportion of 1 must lie between $\frac{5}{16}$ and $\frac{11}{16}$ for a large class of numbers including all the standard mathematical constants such as π, e or $\sqrt{2}$. Details with a computer-assisted proof are in chapter 5, published here for the first time. In another example, I use quadratic dynamical systems on a matrix space with Chebyshev polynomials and self-convolution to uncover beautiful results. Truncated self-convolution of strings and dynamical systems are also core topic in chapters 1– 4. Random polynomials in chapter 7 lead to a spectacular data animation and new developments on the subject. Chapter 9 features new interesting topics related to prime numbers: forbidden constellations, reverse sieving, and magic primes, with a connection to cellular automata.

The style emphasizes simple English even when covering advanced topics, avoiding jargon and advanced mathematics when not necessary. I included enterprise-grade Python code for scientific and high-performance computing with the Gmpy2 library, numerous high-quality illustrations, a comprehensive clickable index and bibliography, along with efficient algorithms not taught in any classroom or textbook. The target audience includes professionals in computer science, physics, AI, machine learning, engineering, quantitative finance, and related fields, as well as students and beginners with one year of exposure to college-level mathematics and Python.

The material opens up new fundamental research areas in theoretical and computational number theory, numerical approximation, dynamical systems, quantum dynamics, and the physics of numbers. It has a strong emphasis on applications: automated pattern detection and theorem proving with AI, agent-based modeling, building a universal unbiased pattern-rich synthetic dataset, cryptography (fast, strong random number generators based on irrational numbers), dynamical systems with chaos detection and isolation, computer-intensive simulations, and high performance computing to handle numbers such as $2^n + 1$ at power 2^n with $n = 10^6$.

Each chapter is self-contained and can be read separately from the others. Most feature results published here for the first time. Chapter 6 and appendix B are new additions and contain a mix of theory, applications, and off-the-beaten path problems with solution. Quantum states and the Riemann zeta function are central themes in each of them. The section on signal processing and discrete convolution is a strong introduction to the topic, serving as a cheat sheet for practitioners or as a solid presentation for beginners, summarizing in a few pages material usually spread over several chapters. Finally, the new chapter 8 discusses a cryptographically secure PRNG that outperforms the most recent addition to the Numpy library, both in terms of speed and randomness. Chapter 9, also a new addition, covers the most recent developments regarding Gilbreath's conjecture, with applications to HPC, cybersecurity, Fintech, PRNGs, chaos modeling, and fraud detection.

About the author

Vincent Granville is a pioneering AI builder, co-founder at Data Science Central (acquired by TechTarget), co-founder and CAIO at Bonding AI, author, patent owner, expert witness and investor included Limited Partner at CalculusVC, a Bay Area VC firm. Vincent worked with Visa, Wells Fargo, eBay, NBC, Microsoft, CNET and several startups. He is also a top AI influencer for NVIDIA and other brands. His AI newsletter has 200,000 subscribers.

Vincent is a former post-doc at University of Cambridge. He published in *Journal of Number Theory*, *Journal of the Royal Statistical Society* (Series B), and *IEEE Transactions on Pattern Analysis and Machine Intelligence* (500+ citations). He is the author of multiple books, available here, including "Synthetic Data and Generative AI" (Elsevier, 2024). Vincent lives in Washington state, and enjoys doing research on stochastic processes, dynamical systems, probabilistic and computational number theory.